Final Exam
MATH 496 Section 2

Instructions: Please work on your own on the problems below. Answer one of the first two problems and four of problems 3-7. Every problem will be worth 20 points. The paper should be handed in by Monday, December 11 by 3pm.

1. Give three derivations of the quadratic formula. For each summarize where you might use it in a classroom. You may assume that certain derivations should only be done in special cases, or that they should only be done after the quadratic equation has been presented some other way. Be as specific as you can, and support your answers with some evidence. You may want to look at high school texts or talk with high school teachers about this problem.

2. Choose one major topic from the class (irrationality proofs, constructible numbers, solving the cubic, algebraic numbers, or Dedekind cuts) and summarize it completely with an outline of the important proofs. Discuss its relevance to the high school curriculum.

3. After discussing the period of fractions with a class, a student asks you the following question: "I noticed that on my calculator, when I multiply any two numbers, then the period of 1 over the product is the larger of the two periods of 1 over the numbers. Why is this true?" (Professorial comment: this is not in general true.)

   Find the smallest product mn for which the students statement is false. Suppose m and n are two positive integers, and suppose that $\frac{1}{m}$ and $\frac{1}{n}$ have periods k and l. State as general a conjecture as you can about the period of $\frac{1}{mn}$. Justify your conjecture as best as you can. This means that if you can, you should prove the conjecture, but if you cannot prove the conjecture, then you should justify it with examples.

4. Create a cubic equation with a rational root for which the general solution method of a cubic given in class gives a solution that does not appear to be rational. Solve it via the method in class, but not using the formula, and then prove that the solution given really is rational. (Your equation should be different from one done in class, the homework, or the notes.)

5. Find a polynomial for which $\sqrt{2} + \sqrt[3]{3}$ is a root. Why should you expect to have gotten a 6th degree polynomial? Explain. Find a basis for $Q[\sqrt{2}, \sqrt[3]{3}]$ over Q. What is the matrix associated to $\sqrt{2}+\sqrt[3]{3}$. Find the inverse of this element expressed as a sum of numbers each of which has a rationalized denominator.

6. Outline how one would construct a segment of length

$$\frac{\sqrt[4]{3+\sqrt{2}}}{\sqrt{5+\sqrt[4]{2}}}$$

given a segment of unit length.

7. Given two Dedekind cuts (A, B) and (C, D), with $(A, B) > 0$ and $(C, D) > 0$, describe the Dedekind cut $(A, B) \cdot (C, D)^{-1}$. Prove directly that this is a Dedekind cut (i.e., don't use that the product of two cuts is a cut or that the inverse of a cut is a cut).