COMBINATORIAL PROBLEMS

A set of boxes are labeled 1 to 6. How many ways are there to put 20 identical balls into these boxes (it is allowed that some of the boxes may be empty)?

A set of boxes are labeled 1 to 6. How many ways are there to put 20 identical balls into these boxes so that none of the boxes is empty?

Let $OP(n)$ be the number of ordered ways of expressing the natural number n as a sum of natural numbers where the ordering counts. For example:

$$3 = 3 = 2 + 1 = 1 + 2 = 1 + 1 + 1$$

so $OP(3) = 4$ and

$$4 = 4 = 3+1 = 1+3 = 2+2 = 2+1+1 = 1+2+1 = 1+1+2 = 1+1+1+1$$

so $OP(4) = 8$. Determine an explicit formula for $OP(n)$.

Consider the set $\{1, 2, \ldots, n\}$. How many subsets are there which do not contain consecutive integers?

How many ways are there to express n as a sum of two numbers where the order of the expression counts? Put another way, in how many ways can you put n balls into 2 boxes if each box must be non-empty?

Starting at the point (0,0) in the plane a move consists of going one unit to the right or one unit to the left. How many distinct paths are there to the point (4,6)? (free th (m,n)?

How many rectangles with lattice points as vertices are there contained in the rectangle with vertices (0,0), (n,0), (0,m) and (n,m)?

a) Suppose that there is a row of n seats and there is a chile in each seat. How many ways are there for the children to reseat themselves if they must occup their original seat or a seat next to their original seat? b) How many permutation π are there of $\{1, 2, \ldots, n\}$ such that for each j,

$$|j - \pi(j)| \leq 1?$$

For a positive integer n define $a(n)$ to be equal to the number of ways of expressing n as a sum of 1's and 2's where the order counts. For example,

$$4 = 2 + 2 = 2 + 1 + 1 = 1 + 2 + 1 = 1 + 1 + 2 = 1 + 1 + 1 + 1$$

so $a(4) = 5$. For an integer $n > 1$ let $b(n)$ denote the number of ways of writing n as a sum of integers each greater than 1, again assuming that order counts and including n itself. For example,

$$6 = 4 + 2 = 2 + 4 = 3 + 3 = 2 + 2 + 2$$

so that $b(6) = 5$. Prove that $a(n) = b(n + 2)$ for all positive integers n.

Consider a circular row of n seats each with a child. The children can rearrange themselves but they can move at most one seat. Let a_n be the number of ways. Determine a_n.

Consider all words of length n made from the letters a, b, c, d. How many have an even number of as?

Let x_n be the number of strings of length n whose elements are from $\{0, 1, 2\}$ and have the property that consecutive terms differ by at most one. Find a recursion satisfied by x_n.