Problems for the first day of class

Magic Square Problem

Fill in the cells of a 3×3 grid with the numbers $1, 2, \ldots, 9$ such that each row, column and diagonal has the same sum. (Free throw).

Discussion

This is an excellent problem to introduce the idea of formulating subgoals, in this case, determining the magic number - the common sum of the rows, columns and diagonals but also the middle square.

Locker Problem

There is set of lockers numbered $1, 2, 3, \ldots, 1000$ and all are unlocked. There are 1000 people standing in a line. The first person then walks by and closes every locker. The second person visits every other locker, starting with 2 and opens each of these lockers. The third person visits every third locker, beginning with 3 and changes its state, that is, locks it if it is unlocked and unlocks it if it is locked. This continues until all 1000 people have gone by the lockers. After the final pass, which lockers are locked?

Discussion

By repeatedly asking he question, "When does an unlocked locker become a locked locker" we turns this into a number theoretic question and thus reformulates the problem.

Ordered Partitions Problem

Let $OP(n)$ be the number of ordered ways of expressing the natural number n as a sum of natural numbers where the ordering counts. For example:

$$3 = 3 = 2 + 1 = 1 + 2 = 1 + 1 + 1$$

so $OP(3) = 4$ and

$$4 = 4 = 3+1 = 1+3 = 2+2 = 2+1+1 = 1+2+1 = 1+1+2 = 1+1+1+1$$

so $OP(4) = 8$. Determine an explicit formula for $OP(n)$.

Discussion

This problem is a good illustration of the value of getting one's hands dirty, of doing several calculations and observing a pattern. Then to establish the conjecture the use of mathematical induction is required.

Equal Differences Problem

Given a set of 8 different natural number $n, 1 \leq n \leq 15$. Show that there are at least three different pairs with a common difference.

Coloring a Chess Board Problem

An "el" is what is left when a corner 1×1 square is removed from a 2×2 square. What is the greatest number of squares of an 8×8 square which can be colored green such that there are no green els?

Discussion These two problems are good introductions to the use of the pigeonhole principle.