GEOMETRY PROBLEMS

Consider a triangle in the plane all of whose vertices are lattice points. Let A denote the area of the triangle, B the number of lattice points on the boudary, and I the number of lattice points in the interior of the triangle. Determine a relationship between A, B and I.

What is the greatest number of regions you can divide the plane into with 3 lines? With 4 lines? With 5 lines? With n lines.

Given n planes in three space let s_n be the greatest number of connected components into which these planes divide the space. Can you find a recursion on n which will allow you to determine a closed formula for s_n? Can you find a combinatorial/extremal argument to determine this number?

Suppose $\emptyset$ is a finite set of point in the plane with the property that the line through any two of them contains as least one other point from $\emptyset$. Prove that all the points of $\emptyset$ are collinear (lie on a single line).

Let $\emptyset$ be a finite set of points in the plane with the property that the triangle formed by any three of them has area at most one. Prove that there is a triangle of area at most four which covers all the points.

Let $\emptyset$ be a set of points in the plane with the property that every point in $\emptyset$ is a midpoint of a line segment joining two points of $\emptyset$. Prove that $\emptyset$ is an infinite set.

Given $2n$ points in the plane, with half colored red and half colored blue prove that it is possible to pair them in such a way that the line segments joining the pairs do not intersect.

Every point in the plane is colored one of three colors: Red, Blue, Yellow. Prove there exists a line segment of length one which has endpoints the same color.

Let $P_1, P_2, \ldots, P_{1999}$ be distinct points in the plane. Draw the line segments $P_1P_2, P_2P_3, \ldots, P_{1998}P_{1999}, P_{1999}P_1$. Is it possible to draw a line L which intersects each of these line segments in an interior point?

Each of ten line segments has length greater 1 but less than 55. Prove that it is possible to select three such that they form the sides of a triangle.

Every point in the plane is colored red or green. Prove that there is an equilateral triangle with its three vertices the same color.