Monochromatic Triangle Problem

Six points are given in the plane, no three collinear. They are joined by all possible line segments (how many?) and each line segment is colored either red or blue (arbitrarily, you don't get to choose the coloring). Show that there must be a set of three points which span a monochromatic triangle.

Friends in Class Problem

There are 30 students in a class. Is the following possible:
9 students have 3 friends in class.
11 students have 4 friends in class
10 students have 5 friends in calls?

Discovering Euler's formula

Investigate some planar graphs and count the number of vertices (V), edges (E), and faces(F) [connected components of the plane left when the edges are deleted]. Find a relationship relating these three numbers.

People at a Party Problem

There are n people at a party, $P_1, P_2, \ldots, P_n$. Let a_j be the number of people with whom person j is acquainted (we do not assume that a person is acquainted with themself and that this relation is symmetric: if P_j is acquainted with P_k then P_k is acquainted with P_j. Prove that there are two different people at the party who are acquainted with the same number of people, i.e. there are $j \neq k$ such that $a_j = a_k$.

Existence of Triangles (3-cliques) in a Graph Problem

Consider a set of $2n$ points in the plane, $n > 1$. Suppose that they are joined by at least $n^2 + 1$ line segments. Show that at least one triangle is formed. Show that for each n it is possible to have $2n$ points joined by n^2 line segments without any triangles being formed.

Integrated Graph Problem

Let $\Gamma = (V, E)$ be a graph. Consider all colorings of the vertices of Γ with the colors black and white. A coloring is said to be **integrated** if each white vertex has at least as many black neighbors as white and each black vertex has at least as many white neighbors as black. Prove that for any graph there always exists an integrated coloring.

Ramsey Problem for Graph on 18 Vertices Problem

Among 18 people there are four who are acquaintances (mutually know each other) or else four who are strangers.

Isolated Vertex in a Graph Problem

Let (V, E) be a graph. For a vertex x denote by $E(x)$ the set of neighbors of $x : E(x) = \{y \in V : \{x, y\} \in E\}$. Assume that whenever $x, y \in V$ and they have the same number of neighbors then they have no common neighbors: If $|E(x)| = |E(y)|$ then $E(x) \cap E(y) = \emptyset$. Prove that there is a vertex x with only one neighbor: $|E(x)| = 1$.