INVESTIGATE INTEGER PATTERNS

Find the sum of the infinite series
$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\ldots+\frac{1}{n\times n+1}\cdots$

Let n be a natural number. Determine the sum of the first consecutive odd natural numbers.

Let $f:\mathbb{B}\rightarrow\mathbb{B}$ be a function which satisfies:

$$f(1)=1, f(2n)=f(n), f(2n+1)=1+f(2n)$$

for all $n\in\mathbb{B}$. Find a simple algorithm for determining $f(n)$.

Consider a triangle in the plane all of whose vertices are lattice points. Let A denote the area of the triangle, B the number of lattice points on the boudary, and I the number of lattice points in the interior of the triangle. Determine a relationship between A, B and I.

Let $OP(n)$ be the number of ordered ways of expressing the natural number n as a sum of natural numbers where the ordering counts. For example:

$$3=3=2+1=1+2=1+1+1$$

so $OP(3)=4$ and

$$4=4=3+1=1+3=2+2=2+1+1=1+2+1=1+1+2=1+1+1+1$$

so $OP(4)=8$. Determine an explicit formula for $OP(n)$.

There is set of lockers numbered $1,2,3,\ldots,1000$ and all are unlocked. There are 1000 people standing in a line. The first person then walks by and closes every locker. The second person visits every other locker, starting with 2 and opens each of these lockers. The third person visits every third locker, beginning with 3 and changes its state, that is, locks it if it is unlocked and unlocks it if it is locked. This continues until all 1000 people have gone by the lockers. After the final pass, which lockers are locked?

What is the greatest number of regions you can divide the plane into with 3 lines? With 4 lines? With 5 lines? With n lines.

Consider the set $\{1,2,\ldots,n\}$. How many subsets are there which do not contain consecutive integers?

Starting at the point (0,0) in the plane a move consists of going one unit to the right or one unit to the left. How many distinct paths are there to the point (m,n)?

Let n be a natural number and $k = [\frac{n}{2}]$ that is, the greatest integer in $\frac{n}{2}$. Determine the following sum

$$\binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \dots \binom{n-k}{k}.$$

Determine all prime p such that $p^2 + 2$ is also prime.

Suppose that there is a row of n seats and there is a chile in each seat. How many ways are there for the children to reseat themselves if they must occup their original seat or a seat next to their original seat?

For a positive integer n define $a(n)$ to be equal to the number of ways of expressing n as a sum of 1's and 2's where the order counts. For example,

$$4 = 2 + 2 = 2 + 1 + 1 = 1 + 2 + 1 = 1 + 1 + 2 = 1 + 1 + 1 + 1$$

so $a(4) = 5$. For an integer $n > 1$ let $b(n)$ denote the number of ways of writing n as a sum of integers each greater than 1, again assuming that order counts and including n itself. For example,

$$6 = 4 + 2 = 2 + 4 = 3 + 3 = 2 + 2 + 2$$

so that $b(6) = 5$. Prove that $a(n) = b(n + 2)$ for all positive integers n.

Consider a circular row of n seats each with a child. The children can rearrange themselves but they can move at most one seat. Let a_n be the number of ways. Determine a_n.

Consider all words of length n made from the letters a, b, c, d. How many have an even number of as?

The set $\Omega_n = \{1, 2, \dots, n\}$ has $2^n - 1$ nonempty subsets. For each such subset multiply the reciprocal of the members and then add all these numbers up. What is this sum? For example, for $n = 3$ the seven subsets are:

$$\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}.$$

The product of the reciprocals for the respective subsets is: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6}$.
The sum is:

$$: 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = 3.$$

Let x_n be the number of strings of length n whose elements are from $\{0, 1, 2\}$ and have the property that consecutive terms differ by at most one. Find a recursion satisfied by x_n.