LOGIC PROBLEMS

Let S be a subset or $\mathbb{R}$ which is closed under multiplication. Let T, U be disjoint subsets of S whose union is S and assume that the product of any three (not necessarily distinct) elements of T is in T and similarly for U. Prove that either T is closed under multiplication or that U is closed under multiplication.

At a party assume that no boy dances with every girl but each girl dances with at least one boy. Prove that there are two couples gb and $g'b'$ which dance whereas g goes not dance with b' and g' does not dance with b.

Find infinitely many solutions to

$$x^2 + y^2 + z^2 = 3xyz.$$