

## LOGIC PROBLEMS

Let  $S$  be a subset of  $\mathbb{R}$  which is closed under multiplication. Let  $T, U$  be disjoint subsets of  $S$  whose union is  $S$  and assume that the product of any three (not necessarily distinct) elements of  $T$  is in  $T$  and similarly for  $U$ . Prove that either  $T$  is closed under multiplication or that  $U$  is closed under multiplication.

At a party assume that no boy dances with every girl but each girl dances with at least one boy. Prove that there are two couples  $gb$  and  $g'b'$  which dance whereas  $g$  does not dance with  $b'$  and  $g'$  does not dance with  $b$ .

Find infinitely many solutions to

$$x^2 + y^2 + z^2 = 3xyz.$$