

## MATHEMATICAL INDUCTION

The Fibonacci sequence is given by  $f_1 = f_2 = 1, f_{n+2} = f_n + f_{n+1}$ . Prove for each natural number  $n$  that

$$f_1 + f_3 + \dots + f_{2n-1} = f_{2n}.$$

2 Let  $f_n$  be the Fibonacci sequence. Prove that  $f_2 + f_4 + \dots + f_{2n} = f_{2n+1} - 1$ .

Let  $f_n$  be the Fibonacci sequence. Prove that

$$\sum_{i=1}^n f_i^2 = f_n \times f_{n+1}.$$

Prove that a set of  $n$  elements has  $2^n$  subsets (including the empty set).

Let  $x$  be a real number such that  $x + \frac{1}{x}$  is an integer. Prove that  $x^k + \frac{1}{x^k}$  is an integer for every natural number  $k$ .

$f_n < 2^n$ . (free throw)

$$f_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right].$$

Consider a  $2^{2000} \times 2^{2000}$  square with a single  $1 \times 1$  square removed. Show that this can be tiled with  $2 \times 2$  ells. An  $m \times n$  ell is a connected collection of squares obtained when a  $(n-1) \times (m-1)$  rectangle is removed from an  $n \times m$  rectangle.

Prove that

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + (n-1) \times n = \frac{1}{3}(n-1) \times n \times (n+1)$$

(field goal). Generalize to

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + \dots + (n-2) \times (n-1) \times n$$

Prove that every natural number can be expressed as a sum of distinct Fibonacci numbers.