MATHEMATICAL INDUCTION

The Fibonacci sequence is given by $f_1 = f_2 = 1, f_{n+2} = f_n + f_{n+1}$. Prove for each natural number n that

$f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$.

2 Let f_n be the Fibonacci sequence. Prove that $f_2 + f_4 + \ldots + f_{2n} = f_{2n+1} - 1$.

Let f_n be the Fibonacci sequence. Prove that

$$\Sigma_{i=1}^{n} f_i^2 = f_n \times f_{n+1}.$$

Prove that a set of n elements has 2^n subsets (including the empty set).

Let x be a real number such that $x + \frac{1}{x}$ is an integer. Prove that $x^k + \frac{1}{x^k}$ is an integer for every natural number k.

$f_n < 2^n$. (free throw)

$f_n = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^n - \frac{1-\sqrt{5}}{2})^n]$.

Consider a $2^{2000} \times 2^{2000}$ square with a single 1×1 square removed. Show that this can be tiled with 2×2 ells. An $m \times n$ ell is a connected collection of squares obtained when and $(n-1) \times (m-1)$ rectangle is removed from an $n \times m$ rectangle.

Prove that

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots (n-1) \times n = \frac{1}{3}(n-1) \times n \times (n+1)$$

(field goal). Generalize to

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + \ldots (n-2) \times (n-1) \times n$$

Prove that every natural number can be expressed as a sum of distinct Fibonacci numbers.