PIGEONHOLE PROBLEMS

Six points are given in the plane, no three collinear. They are joined by all possible line segments (how many?) and each line segment is colored either red or blue (arbitrarily, you don't get to choose the coloring). Show that there must be a set of three points which span a monochromatic triangle.

Given 5 lattice points in the plane prove that you can find a pair amongst them such that the line segment joining them has a third lattice point on its interior (not necessarily from the same set). (Field goal) Generalization: Given 9 lattice points in three space prove that you can find a pair amongst them such that the line segment joining them has a third lattice point on its interior.

Given 12 distinct positive integers prove that there is some pair of them a and b such that either 20 divides a + b or 20 divides a-b.

Prove that there must be two distinct powers of 2 such that their difference is divisible by 37. Use this to show that there is a power of 2, 2^e for some e such that $37|2^e - 1$. What about your proof makes use of 37. Can you extend your solution to prove that if n is an odd number then there is a power of 2, 2^e such that $n|2^e - 1$. Can you extend your proof further to the following let m, n be relatively prime integers. Then there is a power of m, m^e such that $n|m^e - 1$?

Show that among any $n+2$ integers, either there are two whose difference is a multiple of $2n$, or there are two whose sum is divisible by $2n$.

Chose any $(n + 1)$ elements from the set $\{1, 2, \ldots, 2n\}$. Show that this subset must contain two integers which are relatively prime. (free throw)

A group of people are seated around a circular table at a restaurant. The food is placed on a circular platform in the center of the table and this platform can rotate. Each person ordered a different entree and it turns out that no one has the correct dish in front of him or her. Show that it is possible to rotate the platform so that at least two people will have the correct entree.

A chess player perpares for a tournament by playing some practice games over a period of eight weeks. She plays at least one game per day, but no more than eleven games per week. Show that there must be a period of consecutive days during which she plays exactly 23 games.

Every point of the plane is colored either red or blue. Show that there is a rectangle with vertices of the same color.

Every point in the plane is colored one of three colors: Red, Blue, Yellow. Prove there exists a line segment of length one which has endpoints the same color.

Let S be a subset of $\{n \in \mathbb{N} : 10 \leq n \leq 99\}$. Prove that one can find two disjoint nonempty subsets of S with the same sum.

Every point in the plane is colored red or green. Prove that there is an equilateral triangle with its three vertices the same color.