

Problems for 2.18

Choose your favorite integer M and consider the sequence given by recursion: $a_0 = 0, a_1 = 1, a_{n+2} = Ma_{n+1} + a_n$. Note for $M = 1$ this is the Fibonacci sequence.

2/18.1) Prove that if $m|n$ then $a_m|a_n$.

$$\gcd\{a_m, a_n\} = a_{\gcd\{m, n\}}.$$

2/18.3) Can the sum of the squares of 5 consecutive integers be a perfect square?

2/18.4) Let $f(m, n) = 12^m - 5^n$. Determine $\min\{f(m, n) : m, n \in \mathbb{N}\}$.

2/18.5) Let f_n be defined by $f_1 = f_2 = f_3 = 1, f_{n+1} = \frac{1+f_{n-1}f_n}{f_{n-2}}$. Prove t