

### Problems for 2.18

Choose your favorite integer  $M$  and consider the sequence given by recursion:  $a_0 = 0, a_1 = 1, a_{n+2} = Ma_{n+1} + a_n$ . Note for  $M = 1$  this is the Fibonacci sequence.

2/18.1) Prove that if  $m|n$  then  $a_m|a_n$ .

$$\gcd\{a_m, a_n\} = a_{\gcd\{m, n\}}.$$

2/18.3) Can the sum of the squares of 5 consecutive integers be a perfect square?

2/18.4) Let  $f(m, n) = 12^m - 5^n$ . Determine  $\min\{f(m, n) : m, n \in \mathbb{N}\}$ .

2/18.5) Let  $f_n$  be defined by  $f_1 = f_2 = f_3 = 1, f_{n+1} = \frac{1+f_{n-1}f_n}{f_{n-2}}$ . Prove t