Problems for 2/11

2/11.1) A subset S of the real numbers is said to be *sue eree* if it is not possible to find $a, b, c \in S$ such that $a + b = c$. Determine the maximal sum free subsets of $\{1, 2, 3, \ldots, 2n + 1\}$.

2/11.2) a. Suppose there is a row of seats and there is a child in each seat. How many ways are there for the children to reseat themselves if they must occupy a seat adjacent to their original seat?

b. How many permutations π are there of $\{1, 2, \ldots, n\}$ such that for each $j, |j - \pi(j)| = 1$?

2/11.3) Let $n \geq k$ be natural numbers and let $T(n, k)$ be the number of times k occurs in all the ordered partitions of n. For example, suppose $(n, k) = (6, 2)$. The partitions of 6 which include 2 are:

$$2 + 4 = 1 + 2 + 3 = 1 + 1 + 1 + 1 + 2 = 1 + 1 + 2 + 2 = 2 + 2 + 2.$$

The has 2 permutations and has a total of 2 occurances, the second has 6 permutations and a total of 6 occurances, the third has 5 permutations and a total of 5 occurances, the next to last has 6 permutations and a total of 12 occurances, while the last clearly has 3 occurances. This gives a total of 28,

2/11.4) The sequence $a_0, a_1, a_2, \ldots$ satisfies

$$a_{m+n} + a_{m-n} = \frac{1}{2}(a_{2m} + a_{2n})$$

for all nonnegative integers m and n with $m \geq n$. If $a_1 = 1$ determine a_{2000}.

2/11.5) Can the product of 4 consecutive natural numbers be a square?