

Problems for 2/7

2/7.1) At a party assume that no boy dances with every girl but each girl dances with at least one boy. Prove that there are two couples gb and $g'b'$ which dance whereas g goes not dance with b' and g' does not dance with b .

2/7.2) Prove that among 10 consecutive natural numbers there is at least one number which is relatively prime to each of the others.

2/7.3) a. Consider a row of $2n$ squares colored alternately black and white. A legal move consists of choosing a contiguous set of squares (one or more squares but they must be next to each other, no gaps are allowed) and inverting their colors. What is the minimum number of moves necessary to make the row entirely one color?

b. Answer the same question but now start with a $2n \times 2n$ checkerboard and a legal move consists of choosing any subrectangle and inverting its colors.

2/7.4) Prove that there is a term in the Fibonacci sequence which is divisible by 10^{10} . Can you generalize.