Problems for 2/9

2/9.1) Suppose $gcd\{a, 10\} = 1$. Prove that for there is a power of a which when divided by 10^{10} leaves a remainder of 1, i.e. its expression in base 10 ends with 10 zeros and a 1.

2/9.2) Let X be a finite set X with $|X| = n$. Show the subsets of X can be ordered $S_1, S_2, \ldots, S_{2^n}$ in such a way that

a. $S_0 = \{\} = \emptyset$ and

b. $|S_{i+1} - S_i| = 1$, i.e. each succeeding subset is obtained from the previous one by either adding or deleting an element.

2.9/3) Suppose that an integer n is the sum of two triangular numbers:

$$n = \frac{a(a+1)}{2} + \frac{b(b+1)}{2}.$$

Write $4n + 1$ as a sum of two integer squares:

$$4n + 1 = x^2 + y^2.$$

Conversely, suppose $4n + 1 = x^2 + y^2$. Express n as a sum of two triangular numbers.

2.9/4) Given $2n$ points in the plane, with half colored red and half colored blue prove that it is possible to pair them in such a way that the line segments joining the pairs do not intersect.