

Math 30 Introduction to Problem Solving
Week of 3/8

3/8.1) Seven natural numbers have the property that the sum of any six is divisible by 5. Prove that each number is divisible by 5.

3/8.2) Find infinitely many solutions to $x^2 + y^2 + z^2 = xyz$.

3/8.3) An urn contains b black balls and w white balls. Two balls are picked at random and the probability of picking a mixed pair is $\frac{1}{2}$. What are the possibilities for (b, w) ?

3/8.4) Let x_n be the number of strings of length n whose elements are from $\{0, 1, 2\}$ and have the property that consecutive terms differ by at most one. Find a recursion satisfied by x_n .

3/8.5) Let a, b, c, d be integers. Show that the product of the six differences $a - b, a - c, a - d, b - c, b - d, c - d$ is divisible by 12.

3/8.6) Among 18 people there are four who are acquaintances (mutually know each other) or else four who are strangers.

3/8.7) Find all solutions in positive real numbers of the system of equations

$$x_1 + x_2 = x_3^2, x_2 + x_3 = x_1^2, x_3 + x_1 = x_2^2.$$

3/8.8) Let (V, E) be a graph. For a vertex x denote by $E(x)$ the set of neighbors of $x : E(x) = \{y \in V : \{x, y\} \in E\}$. Assume that whenever $x, y \in V$ and they have the same number of neighbors then they have no common neighbors: If $|E(x)| = |E(y)|$ then $E(x) \cap E(y) = \emptyset$. Prove that there is a vertex x with only one neighbor: $|E(x)| = 1$.

3/8.9) Every point in the plane is colored red or green. Prove that there is an equilateral triangle with its three vertices the same color.