

## PROBABILITY

An ordinary deck of cards with four aces is shuffled and then the cards are drawn one by one until the first ace appears. On average, how many cards are drawn?

Given any sequence of  $n$  distinct integers we compute its swap number in the following way. Going from left to right, whenever we reach a number which is less than the first number in the sequence we swap its position with the first number in the sequence. The swap number is the total number of swaps. For example, the swap number of 5,6,3,4,7,1,2 is 2 since we first swap 5 and 3 and then 1 and 3. Find the average swap number of the  $720 = 6!$  different permutations of the integers 1,2,3,4,5,6.

Given a deck of 8 cards. Choose 4 cards at random and put them on the top of the deck in the order in which they originally occurred. What is the average number of cards that are in their original place? For example, if the cards are 1,2,5,7 then the new order is 1,2,5,7,3,4,6,8 and 1,2,5,7 are in their original place. How about 10 cards and five are chosen? How about choosing  $n$  cards from a deck with  $2n$  cards?

An urn contains  $b$  black balls and  $w$  white balls. Two balls are picked at random and the probability of picking a mixed pair is  $\frac{1}{2}$ . What are the possibilities for  $(b, w)$ ?