You have all the elements here and since I know what you are doing I can dig through it and see that there is, indeed, a complete proof modulo the fact that you misuse the notation. It is a good idea at the beginning of solution to define things and introduce notation. Here are some ideas that may help you get this written up rigorously:

$\Omega_n = \{1, 2, \ldots, n\}$. Denote by P_n the power set of Ω_n (the collection of all subsets), and by $\hat{P}_n$ the non-empty subsets. For a non-empty subset S let $\frac{1}{S}$ denote the product of the reciprocals of the elements in S. Now what you are trying to prove is

$$\Sigma_{S \in \hat{P}_n} \frac{1}{S} = n.$$

The initial case is $n = 1$ and this is obvious. The inductive hypothesis is the statement assume we have shown for n that

$$\Sigma_{S \in \hat{P}_n} \frac{1}{S} = n$$

then we must show

$$\Sigma_{S \in \hat{P}_{n+1}} \frac{1}{S} = n.$$

Now partition $\hat{P}_{n+1}$ as

$$\hat{P}_n, \{n+1\}, \{S \cup \{n+1\} : S \in \hat{P}_n\}.$$

Good luck.