

2/28.2) The investigation and conjecture are good but you are really having a problem with induction. First of all you need to state fully the inductive hypothesis:

Let  $\Omega_n = \{1, 2, \dots, n\}$ . For each non-empty subset of  $\Omega_n$  multiply the reciprocals of its elements and add these up. Then the sum is  $n$ . Now you want to show that the next case holds. But where in your proof do you show that when you find these products for the subsets that are not in  $\Omega_n$  and add these up that this difference is 1? This is the crux of the problem. You dealt with it by just saying it is so. But why? Try actually doing some examples. What you should notice is that when you pass from one integer, say  $n = 4$  to  $n = 5$  you get 16 additional subsets. One of these is just  $\{5\}$  with reciprocal  $\frac{1}{5}$ . All the other are of the form  $S \cup \{5\}$  where  $S$  is a non-empty subset of  $\Omega_4$ .

Some appropriate notation might help. Let denote by  $P_n$  the power set of  $\Omega_n$  (the collection of all subsets), and by  $\hat{P}_n$  then non-empty subsets. For a non-empty subset  $S$  let  $\frac{1}{S}$  denote the product of the reciprocals of the elements in  $S$ . Now what you are trying to prove is

$$\sum_{S \in \hat{P}_n} \frac{1}{S} = n.$$

Now in passing from  $\Omega_n$  to  $\Omega_{n+1}$  there are an additional  $2^n$  nonempty subsets, precisely those containing  $n + 1$  and  $\hat{P}_{n+1}$  is partitioned by the following subsets:

$$\hat{P}_n, \{n + 1\}, \{S \cup \{n + 1\} : S \in \hat{P}_n\}.$$

The inductive hypothesis states that

$$\sum_{S \in \hat{P}_n} \frac{1}{S} = n.$$

So you need to show that

$$\frac{1}{n + 1} + \sum_{S \in \hat{P}_n} \frac{1}{S \cup \{n + 1\}} = 1.$$

Good luck.