2/28.2) Nice investigation and conjecture. Induction is the way to go but you havent quite executed it correctly. You have the base case correct and the inductive hypothesis. Sometimes before proceeding it is good to introduce some notation. Let $\Omega_n = \{1, 2, \dots, n\}$ for each natural number n. Let P_n be the power set of Ω_n, that is the collection of all subsets and $\hat{P}_n$ the collection of all nonempty subsets. For a non-empty subset S let $\frac{1}{S}$ denote the product of the reciprocals of the elements in S. Now what you are trying to prove is

$$\Sigma_{S\in\hat{P}_n}\frac{1}{S} = n.$$

Now in passing from Ω_n to Ω_{n+1} there are an additional 2^n nonempty subsets, precisely those containing $n+1$ and $\hat{P}_{n+1}$ is partitioned by the following subsets:

$$\hat{P}_n, \{n+1\}, \{S \cup \{n+1\} : S \in \hat{P}_n\}.$$

The inductive hypothesis states that

$$\Sigma_{S\in\hat{P}_n}\frac{1}{S} = n.$$

So you need to show that

$$\frac{1}{n+1} + \Sigma_{S\in\hat{P}_n}\frac{1}{S \cup \{n+1\}} = 1.$$

Now the inductive hypothesis is that

$$\Sigma_{S\in\hat{P}_n}\Pi_{s\in S}\frac{1}{s} = n$$

See if you can use this to show that when you take all the elements in the second set and for each one multiply the recipocal of its elements and add up then you get