RECURSION

Given a convex polygon in how many ways is it possible to divide it into triangles whose vertices are the vertices of the polygon?

Let n be a positive integer and define

$$f(n) = 1! + 2! + \dots n!$$

Find polynomials $P(x)$ and $Q(x)$ such that

$$f(n+2) = P(n)f(n) + Q(n)f(n+1).$$

Choose your favorite integer M and consider the sequence given by recursion: $a_0 = 0, a_1 = 1, a_{n+2} = Ma_{n+1} + a_n$. Note for $M = 1$ this is the Fibonacci sequence.

Prove that if $m|n$ then $a_m|a_n$.

For m, n positive integers prove that $gcd \;\; a_m, a_n \;\; = a_{gcd\{m,n\}}$.

Let f_n be defined by $f_1 = f_2 = f_3 = 1, f_{n+1} = \frac{1+f_{n-1}f_n}{f_{n-2}}$. Prove that f_n is a natural number for all n.

Find infinitely many solutions to

$$x^2 + y^2 + z^2 = 3xyz.$$

Find infinitely many solutions to $x^2 + y^2 + z^2 = xyz$.