Problems for the second week of class

Monk Problem

A monk climbs a mountain. He starts at 6am andd reachs the summit at noon. He spends the night on the summit. The next morning he arises early and leaves the summit at 6am and descends by the same route he used the day before and reachs the bottom at noon. Prove that there is a time between 6am and noon at which the monk was at exactly the same spot on the mountain on both days.

Discussion

This is a good example of adding auxilary elements and can involve creativity.

Some pigeonhole problems

Finding a monchromaic Triangle

Six points are given in the plane, no three collinear. They are joined by all possible line segments (how many?) and each line segment is colored either red or blue (arbitrarily, you don't get to choose the coloring). Show that there must be a set of three points which span a monochromatic triangle.

Divisibility by 10

Given 12 distinct positive integers prove that there is some pair of them a and b such that either 20 divides a + b or 20 divides a-b.

Chess Player Problem

Circular Table Problem

A group of people are seated around a circular table at a restaurant. The food is placed on a circular platform in the center of the table and this platform can rotate. Each person ordered a different entree and it turns out that no one has the correct dish in front of him or her. Show that it is possible to rotate the platform so that at least two people will have the correct entree.

Discussion

These problems are specifically introduced or assigned to give students he opportunity to work on accessible problems which can be solved using the pigeonhole principle.

Infinite Series Problem

Find the sum of the infinite series
$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \ldots + \frac{1}{n\times n+1} \cdots$

Discussion

This is another problem where a student can do an investigation in order to get a sequence of values and find a pattern. To finish the problem off requires a proof by induction and this is a good entry to this proof method.

Fibonacci Problems

The Fibonacci sequence is given by $f_1 = f_2 = 1, f_{n+2} = f_n + f_{n+1}$. Prove for each natural number n that

$f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$.

Let f_n be the Fibonacci sequence. Prove that $f_2 + f_4 + \ldots + f_{2n} = f_{2n+1} - 1$.

Let f_n be the Fibonacci sequence. Prove that

$$\Sigma_{i=1}^{n} f_i^2 = f_n \times f_{n+1}.$$

Discussion

The Fibonacci sequence is a good source of induction problmes.