Name:

Math 496 Section 2
Midterm

1. State the definition of what it means for a real number α to be irrational. Give one other equivalent characterization (10 points)

2. State the Dimensions Multiply Theorem. Be sure to include the hypotheses. Explain how this relates to bases for vector spaces. (10 points)

3. Suppose α is a real number, and that for all n there exist integers A_n and B_n such that $0 < A_n\alpha + B_n \leq \frac{1}{n}$. Show that α must be irrational. (15 points)

4. Prove that $\sqrt{52}$ is irrational. (15 points)

5. Using straightedge and compass, construct a segment of length $\sqrt{3}$ given a segment of length 1. In addition, give the steps in the process. (15 points)

6. Consider the field $Q[\sqrt{3}]$, and the basis $e_1 = 1$ and $e_2 = 1 + \sqrt{3}$. What is the matrix for the element $2 + \sqrt{3}$ under this basis. Express the inverse of this element in terms of e_1 and e_2. (15 points)

7. Outline the proof that you cannot construct $\sqrt[3]{2}$ with straightedge and compass. You should include all of the major steps, starting with the statement that the constructible numbers form a field. Thus you should explain what major results are needed to show this result, and not simply quote one theorem, but rather outline how that theorem is proven. (20 points)