

## Study Guide

The test will consist of a few problems similar to those done in class or on the homework, in addition to one or two problems using concepts from the class. One of these problems will probably be an essay problem in which you will have to put organize material from the class. One problem might also ask you to use ideas developed in the class in new ways. To prepare for the test, it is advisable that you read all of the solution sets and the comments in them, in addition to reviewing your class notes and homework.

The main topics that we have covered include:

- Definition and operations with rational numbers.
- Information about the decimal representation of rational numbers: (period, relationship to powers of 10, relationship with the division algorithm, etc.)
- Irrationality proofs for radicals.
- Irrationality proofs for  $e$  and  $\pi$ .
- Comensurability.
- Geometric constructions and the field axioms.
- Extension fields as vector spaces and representing them as matrices.
- The impossibility proofs.

You should be able to state important definitions from each of these topics, and you should feel comfortable with the main ideas from each of these topics. You should also be able to use these ideas and apply them to high school algebra, geometry, and trigonometry problems, where appropriate.

Below are some sample problems.

1. Suppose that  $p$  is a number with the property that  $10^{12} - 1$  is divisible by  $p$ . What can you say about the period of  $p$ ? Give an explanation of why this is true that would be appropriate for a 16 year old algebra 2 student.

2. Explain what it means for two lengths to be commensurable, relate this definition to the measurements of the lengths.
3. Explain how to construct a thirty degree angle.
4. Prove that  $\sqrt[3]{24}$  is irrational.
5. To rationalize the denominator of  $\frac{1}{2-\sqrt[3]{5}+2\sqrt[3]{25}}$ , what should you multiply the top and bottom of the fraction by?
6. What property of rational numbers do we (contradict) in the geometric proof that  $\frac{1+\sqrt{5}}{2}$  is irrational. Explain why this property is true.
7. Suppose you had construction tools that allowed you to construct a second, third, or fifth root of a previously constructed number. Suppose further that the only field extensions that could be obtained by single construction steps were of degree 2, 3, and 5. Which of the following numbers could be constructed with these tools:
  - (a)  $\sqrt[3]{2}$
  - (b)  $\sqrt[6]{2}$
  - (c)  $\sqrt[7]{2}$

Explain your answers.

8. Two fractions  $\frac{a}{b}$  and  $\frac{c}{d}$  are equivalent if and only if  $ad = bc$ . Explain how this equivalence relation forces the definition of fractional addition given by  $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$  to be incorrect.