

# Math 496 Homework

Due Friday, Nov. 10.

1. Show that  $\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}}$  is a rational number. Hint: Find rational numbers  $a, b, c,$  and  $d$  such that  $(a + b\sqrt{3})^3 = 26 + 15\sqrt{3}$  And  $c + d\sqrt{3} = 26 - 15\sqrt{3}$ .
2. Find a polynomial with rational coefficients that  $\sqrt[3]{26 + 15\sqrt{3}}$  is a root of.
3. Solve with radicals by hand (showing your work) the equation

$$x^3 + 6x = 12.$$

4. Solve with radicals by hand (showing your work) the equation

$$x^3 + 3x^2 + 15x + 3 = 0.$$

5. It is unknown whether  $e\pi$  and  $e + \pi$  are transcendental or not. Curiously, it is known that at least one of them must be transcendental. Use the following outline to prove this:
  - (a) Show that either  $e + \pi$  or  $e - \pi$  is transcendental. (You are allowed to use that  $e$  and  $\pi$  are transcendental, and that the algebraic numbers form a field.)
  - (b) Calculate  $(e + \pi)^2 - 4e\pi$  and factor.
  - (c) Using that the algebraic numbers are closed under square roots, show that if both  $e + \pi$  and  $e\pi$  are algebraic then  $e - \pi$  would also be algebraic.
  - (d) Using the first part, show that either  $e + \pi$  or  $e\pi$  is transcendental.
6. Discuss the difference between transcendental and algebraic numbers and how this affects a student's ability to understand each. How might this influence your teaching about  $\pi$  and  $e$ ?