Ready or Not

"Algebra for everyone" is the policy at the large suburban San Francisco high school where I student-teach in the Mathematics department. This means that Algebra I is not only a required course, but it is also the lowest level mathematics course offered at the school. No alternative courses are available for those who might not be fully prepared for Algebra when they enter as freshmen, and no senior can graduate without a passing grade in Algebra.

One practical result of this policy is that my fall term class of 22 students included eleven freshmen taking Algebra for the first time, and eleven upperclassmen who had failed it one, two, or even three times in prior years. I quickly learned that some freshmen were indeed not ready for this course, in that they lacked basic skills such as computations with fractions and had conceptual difficulty in multiplying with negative numbers. The most mathematically talented members of the freshmen class were unrepresented here, because they had completed Algebra I in middle school, and thus bypassed this course completely. Several of the upperclassmen had obviously not been ready on earlier attempts, and repeated efforts to master the Algebra course content had not addressed their pre-existing gaps in skills and concepts. Ready or not, Algebra was what they were to learn in my class.

My first impression of this group was that they were remarkably quiet. Some of this was probably due to the 7:30 a.m. starting time of the class, made more onerous by the hour or longer bus rides that some took to get to school. The block schedule adopted by the school put them in my class for 90 minutes each morning, five days a week, as the start to a very long day. There was much material to be ready for and to master each morning, because the entire Algebra course was to be completed between September and January.

The class was an ethnically diverse group, in which girls outnumbered boys sixteen to six. I was relieved to find that all were competent in spoken English; as a beginning teacher, my tasks seemed formidable enough without the added complications of language issues. The class was well behaved, presenting few management challenges; indeed, I wished they would be more talkative and active, so that I could exploit that energy for class discussions.

Initially, I would begin each day with a set of four to six warm-up problems posted on the board to get them ready for the day. Usually, these were chosen to explore or illustrate a single theme or property, drawn from the prior day's material. The intent was to get the kids thinking about math, wake them up, review the prior content, and add a bit of insight to what had been done the previous day. Students worked on these problems individually, and then volunteers would put solutions on the board for class discussion. After the warm-up, a new lesson would be started. For the lessons, I often placed the class in groups of four, chosen to ensure a mix of ability and to honor student preferences for particular work-mates as much as possible.

Early on, I began to notice a pattern in the performance on warm-up problems and quizzes. When I assigned a purely computational problem, or one requiring symbolic manipulation to solve an equation, most students would usually attempt the problem, and some would correctly answer it. However, when I devised a problem written in English, perhaps asking for an application of a mathematical principle or technique, it seemed to stop the whole class dead in their tracks. When faced with a "word" problem, they would freeze up, with stunned facial expressions, reminding me of a surprised deer caught in the beam of my car headlights.

I was puzzled at first as to why these problems appeared to be so hard for the class. A problem already written in mathematical notation, or an equation already written, seemed well within their grasp. But a word problem that could be represented by the *same notation or equation* brought them to a screeching halt, even when it was given as the very next problem.

I thought hard about what the possible causes might be. The most obvious possibility was difficulty with basic reading skills. I soon decided that some sort of experiment would be necessary to explore or eliminate that possibility. I gave the class moderately complex written instructions for problems written in equation form. They seemed to have no trouble understanding the instructions, and took the equations in stride. I also tried a few more word problems in class, asking selected students to read them aloud from a workbook. Reading did not seem to be the issue.

I also considered the possibility that these students were unaccustomed to being asked to think in a math class, and simply refused to do so. As I continued to sprinkle word problems into daily warm-ups, I watched their faces and listened carefully to the few questions that this quiet group would ask. However, I saw no defiance, only bewilderment.

I concluded that the difficulty must lie in recognizing the mathematical content in a problem statement, and then generating a translation into mathematical notation. In effect, they did not know how to get started. Once an equation was in hand, they could solve it, but they couldn’t see how to begin. I realized that this would cripple them for any higher mathematics, and relegate any math they already knew to an academic curiosity, with no hope of practical application. Although I did not know it at the time, I had run smack into one form of a pervasive problem that Howard Gardner calls “rigidly applied algorithms”. As Gardner puts it, “Only when the problem as set actually triggers the algorithm that has been mastered will students get the correct answer; as soon as there is any alteration in the formulation of the problem, the student is likely to get completely lost.” Gardner, *The Unschooled Mind* (1991), New York: BasicBooks, at p. 166. This is a subspecies, peculiar to mathematics, of the more general problem of lack of transfer of knowledge. Students who can handle all the algebra necessary to solve a problem in the abstract are unable to cope with its application in a realistic setting. See Bransford et al. (Eds.) (1999), *How People Learn: Brain, Mind, Experience, and School,* at pp. 61-65.

I decided to start a personal quest to cure the word problem phobia for these kids. I saw their phobia as a simple consequence of yet another gap in their readiness for Algebra, and as one that I could address directly and promptly. I surmised that no prior math teacher had taken the pains to prepare them, by arming them with strategies for solving problems of this type.

As a first step, I resolved to put at least one word problem into each daily warm-up. It was rare to get a full solution from a student, but I would always be sure to provide one if the class could not. When we reviewed the problems, I would explicitly talk about and thoroughly question the solution strategy that applied, whether the source of the solution was a student or me. I thought that practice would help, and that explicit discussion of strategy might demystify the whole subject. In my own naive way, I was attempting to reinvent the cognitive apprenticeship approach, making each example of successful thinking visible to the class. See e.g., Collins, Brown, and Holum (1991) "Cognitive Apprenticeship: Making Thinking Visible" in *American Educator*, (AFT, Winter 1991) at p. 9.

To prevent students from simply "shutting down" when they saw a problem written in English, I modified the procedure for the warm-ups. I created a new rule that nobody was allowed to give up on a problem. If they were stumped, they were required to get up from their seats and find some other students to talk to about it. If a pair or more were stumped, then they could come to me together for a hint.

These measures did not produce immediate improvement, so I continued to work on ways to help the class over this hurdle by directly attacking the readiness issue. I surmised that modeling, making thinking visible, and providing peer assistance might not be enough if I continued to present problems of the same complexity and difficulty. To make them truly ready, I needed to break the task down into smaller parts, and provide some scaffolding (yet another element of cognitive apprenticeship). To that end, I began some training in writing equations from English statements. I showed the class some fundamental ideas, such as converting "is" to "=", and "and" to "+". We also considered more complicated phrases such as "four more than a number" (x+4) and "reduced by 25%" (–.25x). A typical exercise in the early stages was to re-write "the width is four more than the length" as "W=L+4". We used a variety of common formulas, such as distance equals rate times time (d=rt), age equals grade level plus six (a=g+6), and so forth.

I discussed my quest with my Cooperating Teacher, who whole-heartedly approved of my efforts, and shared my view that readiness of the class was the central issue. He offered me the use of a large set of very similar problems that he had prepared many years before in a now defunct pre-Algebra course, intended to address general readiness for Algebra explicitly. He also suggested an additional problem type, in which the class was asked to make up English statements from equations, running the solution process in reverse. More students seemed to be attempting these each day, and correct answers became more and more frequent.

I also introduced a new form of problem. I would frequently ask the class to write an appropriate equation from a written statement, *without* then asking them to solve the equation they had just written. This seemed sensible, because the difficulties all seemed to lie in the initial translation steps. It also allowed me to use a wider variety of problems. For example, I could pose area computation problems, which sometimes led to quadratic equations. The class had not yet been taught to solve quadratics, so they could not carry through the later steps, but they had the requisite knowledge to read and write such equations. I thought that this writing-without-solving effort might send the important message that mathematics lies not only in getting numerical answers but also in modeling situations, without regard to whether one can solve the problem thus described.

I worked a bit of this into each day's activities. Lessons continued on the normal curriculum, as I tried to fold this content in where I could. Eventually, I also shifted to starting the class in groups, so that the warm-up problems could be done as collaborative efforts. I asked the class what they thought would help them. Troy, a poor math student but a natural leader, suggested that different groups should get different problems to work on, to add variety and ensure full participation by preventing one group from piggy-backing on the efforts of another. I adopted that suggestion, too.

I did no formal assessment on word problem mastery, instead relying on my impression that participation in solving the warm-up problems was increasing, and judging from the warm-up discussions, comprehension appeared to be increasing as well. To demystify the subject as much as possible, I prepared a short handout with tips for getting started, that was discussed in class and applied to some sample problems. Although I was then unaware of the work of Collins, et al., in their terminology, I was suggesting heuristic strategies. Collins (1991). at p. 39. The handout looked like this:

HOW TO GET STARTED WITH WORD PROBLEMS

1. Figure out what you're looking for (a price, a length, etc.).

2. Give it a name (a variable, such as X or L or P or ....)

3. See if you can write the other quantities in the problem in terms of the variable. (L+2, or _ X, or .07P, etc.)

4. Look at the problem to see what the overall relationship is. Do several lengths add up to a total? Do several prices add up to a cost? Do two

sides multiply to make an area?

5. Write out the relationship from step 4 using the expressions you wrote at step 3.

My lingering concern was that all this might go from terrifying to boring without having passed through understanding along the way. To minimize boredom, I decided to create a personalized set of problems that might have more interesting content. I based this on what I knew of the hobbies and interests of many students in the class. I knew that we had several junior varsity athletes in girl's volleyball and tennis, several devoted fans of a popular teen-idol band, and one girl who aspired to a singing career. I prepared one custom problem for each group, based upon the individuals in it. I put considerable effort into including the kids' names in the problems, and researching a good basis for each problem setting (for example, one dealing with concert tickets reflected the actual ticket prices for the next major concert booked by the band in question, that I had determined by online research) that would produce nice whole number answers.

Each problem required writing and then solving a linear equation. Two samples are reproduced here:

Kristin buys two tickets for the upcoming Reno concert of 'N-Sync, by calling the B.A.S.S. telephone service. B.A.S.S. adds $6 per ticket service charge. Michelle buys two tickets through Ticketmaster, and they add 20% as a service charge.

They spend a grand total of $188. What is the ticket price?

Brandee gets a recording contract, so Angela and Laurie decide to buy her first cd. Angela picks up one for Cathy, too. Laurie shops online with amazon.com and they charge 9% for shipping and handling. Angela goes to Tower Records and has to pay 8% tax. If they spend a total of $32.50, what's the price of the cd?

After three weeks of practice, explaining strategies, showing translations tips, providing handouts, and collaborative work on problems of this sort, I expected all to go well when I presented this set of problems to the groups. I thought that this would be a good measure of the success of my quest. I had convinced myself that they were ready.

When class began the next day, I handed out the problems with high hopes. I could see the students initially reacting with surprise to see their names in the problems, and could overhear comments such as "How did he know I play tennis?". The problems were not difficult, in my estimation. I anticipated that allowing for a bit of inefficiency in group communication, some groups would have solutions fully worked out in under ten minutes, and all would have answers within fifteen.

However, as the clock ticked on past ten minutes, then fifteen, and then twenty, I became more and more worried. Most groups were stumped. I reminded everyone that they could use their "how to get started" handouts, and some renewed their efforts. After twenty-five minutes, one group had reached a solution. I waited another five, but no other group had meaningful progress to report. I asked the successful group to present their solution, and explain to the class how they obtained it. I suggested to the others that they take the problem home and consider it overnight, given whatever added insight they may have obtained from their peers' presentation. The next day, there were no additional solutions to present.

At this point, I had a much stronger sense of failure than any of the students. They, at least, were accustomed to being stumped by problems of the sort they were facing. I was not accustomed to being stumped by the problem I faced. I had tried everything I knew, and it obviously wasn't enough. I reluctantly decided that my personal quest was sapping too much time and energy, and that I would have to abandon it for the foreseeable future. It was time to move on to new content, ready or not.

As I look back on this experience, I think that my choice of a personal quest on this topic was a good one, because it presents a hurdle that might prevent mathematics from ever coming alive or being truly useful for these students. I would even have been content to have my students forget most of the specific techniques taught in the course if they could only master this one difficulty.

I suspected that many years of prior bad experience in math classes had left these kids ill-equipped to face mathematical problems that required thinking and application rather than just mechanical manipulation and computation. I did not, however, attempt to track down what had actually happened in their prior classes. Was this a case of hubris on my part, assuming inadequacy of prior teaching that I, a student teacher, presumed to correct quickly? This should be the sort of material that is included in a Bruner-style spiral curriculum, in which solving real problems is approached in an intellectually honest way at every grade level, with increasing complexity as students mature. See Bruner, J. (1960) *The Process of Education* at p. 33. Perhaps prior teachers had addressed this, and I misunderstood the situation. Did my own conceptualization of appropriate problem solving processes lead me to mis-diagnose the underlying problem suffered by the class?

Alternatively, I may simply have failed to be a good master to these cognitive apprentices. As I struggled with the problem, I continually modified my approach, to provide more help at each stage as my perception of what might be necessary evolved. I began with additional practice, added modeling of strategy, then tried scaffolding with smaller "chunks" of problems, then added handouts for getting started, and finally, I tried spicing up interest in the problems. This process of continually stepping back may have caused me to use these tactics in the reverse order. Perhaps if I had started my campaign at the most basic level, and had provided a coherent path building upward from there, the apprenticeship would have been more sensible to the apprentices. I think that the basic components I used all have value, but their coordination is another matter. Furthermore, feeling the pressure for course "coverage," I also abandoned the quest after only three weeks. Even on the double-pace block schedule, that makes for a very short apprenticeship, allowing little time to overcome entrenched ways of thinking.

Finally, I have realized that when I dove into this problem, I did so *ready or not.* I did not research it, beyond asking my Cooperating Teacher and my class for their views. I did not consult the literature. Since this subject matter was not addressed directly in the standard textbook and materials for the Algebra course, I took it upon myself to create the materials and the pedagogical approach from scratch. To that extent I fell into the same trap as Virginia White, in her case “One Struggle After Another” (in J. Shulman & J. Colbert (Eds.) *Intern Teacher Casebook*; San Francisco: Far West Lab). Others have certainly faced this difficulty before. Phobias regarding word problems are common, so much so that I have even heard comedians begin a joke with, “If two trains leave Chicago at 3:14 p.m. . . . .” I set myself the task of reinventing, and that is a formidable challenge for a new teacher.

Ultimately, this is a case of failure to transfer skills of symbolic manipulation to realistic situations in which those same skills would solve the problem. The students were not ready to succeed at that transfer, and I was not ready to coach them to success.