Enriching the Mathematical Experiences of All Students
Through the Routine Use of the Rule of 3
TRG Revised Case
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I. Rationale

My rationale has changed little since I first wrote my case. However, I now can add a few points. I would add that almost every single text that our district is looking at for adoption next year has far stronger algebraic and representational components than our current texts *(see attached pilot text samples*). Furthermore, use of the Rule of 3 can be seen as a way to provide students with multiple strengths new ways to access the mathematical topics that we are studying (a la multiple intelligences, see Josh's and Debbie's forthcoming cases for more complete information). Finally, the Rule of 3 can facilitate connection among math concepts and opportunity to routinely revisit topics that we have explored.

To refresh your thinking about my case my rationale from last time:

Simply put the Rule of 3 states that every topic in mathematics should be presented numerically, algebraically, and geometrically (I understand that this last term refers for the most part to graphic representation). I have multiple reasons for working to understand the instructional and learning implications of the Rule of 3. First, there are several parents who inquired, in some cases forcefully, about what would be done to challenge their highly capable children. The students in fifth grade this year do indeed represent probably one of the most pronounced splits between highly capable and struggling students (with very few students in the middle) that we have had in quite some time. This seems to accentuate the needs of both groups. Exploring this strategy may be a way to address the needs of the mathematically advanced students, while not creating a whole new curriculum for them. Second, for the past few years I have noticed that the mathematical support that students are able to give for their problem solving solutions is often minimal or altogether lacking. Since I have never done a TRG case in mathematics, this topic provides a unique opportunity to match a real need with a personal change of pace. Third, in some senses given the direction of our district this year a case into extending learning experiences for otherwise "MEAP" ready students might seem like I am pointing my arrow in the wrong direction. However, since the tests that students take are so heavily grounded in multiple representations and since I am attempting to make my work on the Rule of 3 accessible to most of my students my work might seem more justifiable.

II. Information on Instructional Practices

Each week I give a set of five math problems to my students that center on the math content that will be taught during the week *(see instructional examples)*. Students choose to complete a minimum of three problems from the "menu" and work independently, with small groups, and through large group discussions to solve these problems. My approach to encouraging the use of the Rule of 3 has three components. I still do not have this down to any sort of science (in fact if the group could help me think of a more systematic way to do this work that could be very helpful). First, during menu time I observe the progress of each student and get a feel for the sorts of mathematical work they are doing. When a student is at a point where a Rule of 3 prompt would help him/her to think more productively or thoroughly I engage them in one-on-one or small group discussions. It is not all that easy to describe how I know when to intervene, but examples include when students are "stuck" with particularly unproductive methods or when students are "done", but have not thought very deeply about their work. Second, at the end of the week we have whole group discussions of menu problems. Usually students select one problem that they would like to discuss and I also select one. For my selection I often choose a problem with the most Rule of 3 potential. Sometimes I "plant the seeds" of different representations and then I "harvest" those during the discussion. I attempt to show students how the Rule of 3 can provide insight into their work. I am trying to show how these different representations are useful in problem solving, not just a novelty or busy work. I am encouraging students to think about the ways that these representations help them to see different aspects of the problem or understand the possibilities of the solutions better. Third, I have created my own tasks (and used math text pilot materials) that lend opportunities to teach lessons that show how graphs and equations relate to 5th grade curricular topics *(see instructional examples and additional readings).*

Most of the time when I am teaching about the Rule of 3 I present a situation where students can generate many solutions and then look for patterns in those solutions that would be interesting or useful to show in graphs and equations. Basically I am teaching through induction, but I am starting to wonder if I am missing the boat by not using the equations/graphs to do some deduction too. For example I just tried to have students develop the formula for the sum of angles in triangles (adding all angles produces a sum of 180) and quadrilaterals (adding all angles produces a sum of 360). Maybe I could have had them extend this deductively to other polygons. For instance, the sum of angles in an octagon is T x 180 where T = the number of triangles inside the octagon OR (N-2) x 180 where N= the number of sides on the shape. Just a note: This problem may seem like it centers on a fringe topic, but students who explore it are developing skill in measuring angles, using operations (+, x, …), learning about the properties of shapes, as well as Rule of 3 related skills.

III. Case Insights

In my rough case I shared a few tentative insights about my lack of knowledge of Rule of 3 connections to math menu problems and how students learn to use variables. I also wrote about the broad way in which my students' work showed the graphic part of the Rule of 3 and how the Rule of 3 actually can be thought of as having a strong communication component. At this point I can add a bit to this and also have a few new insights.

The last time we met I shared how some Rule of 3 representations were difficult to generate for certain math problems. There have been some problems since then that have fit all Rule of 3 components very well (*see instructional tasks and work samples from Menu 3.3 problem 5*). In most cases I have pretty much given up on pushing students to consider every representation for every problem. Instead I think it has been more productive to consider which representations will really highlight something important while also being manageable for the students. In order to make decisions about which representations to stress I have had to explore menu problems and assignments myself with a Rule of 3 frame of mind. For instance, one assignment that I have included here explores the relationship between different attributes of 3D shapes *(see instructional tasks and work samples)*. While it is possible to use graphs to look at the relationships, that sort of representation is not all that informative. However, creating equations that show these relationships is very interesting and also productive for thinking more deeply about the shapes themselves.

Last time I was concerned that students were trying to make everything a variable. In some cases this is a problem, but in two ways this might not be as big of an issue as I first thought. First, I think that when students make everything a variable that is a sort of first level analysis of what is happening in the problem. This is not where I want students to end up, but it gives me more information about how students are thinking about the problem and may help them to clarify what is going on. Second, in some mathematical strands, like geometry, many equations DO have multiple variables. For example $a^2 + b^2 = c^2$ or the equations that my students worked on with Faces, Vertices, and Edges *(see instructional and work samples)*. So the point isn't to always work toward one variable, but toward a number of variables that can help students to make sense of their mathematical work and that fit with the mathematical strand that we are focusing on.

One of the things that is still bothering me is that when students explore the different representations it sometimes isn't very clear what they are thinking or what they mean to show. This is where I have to do some more thinking about how I will encourage the kids to communicate more about their Rule of 3 explorations. I don't just want to see a bunch of letters, but rather some kind of explaining or labeling about what the letters mean or why they are writing an equation in a certain way. However, if I require too much "extra" explaining it might deter students from using these representations. It also might seem like overkill. After all these representations are supposed to communicate, not just be something that needs to be communicated about (if that makes any sense).

To study student's use of the Rule of 3 and my instruction related to it I have been video taping lessons and looking at student menus. The video works well in whole group discussions, but as you all know sometimes tapes capture the mundane and not the revealing/interesting. Some really interesting case related lessons happened when the tape was not rolling (ex. The F+V-2=E lesson *see student work samples*) With student work I am noticing more regular use of Rule of 3 ideas, even with students that I do not encourage in one-on-one fashion. As I wrote earlier, some students who are experimenting are not very clear about what their graphs or equations show.

IV. Questions for the Group

1) Now that you have read about my instructional approach, does my "pick and choose" approach about who to work with and when to work with them make sense OR does it leave too much to random chance and have other drawbacks that I am not yet considering?

2) What do you think about the adjustment to my menu assessment tool? Can you think of another way to emphasize the importance of multiple representation without limiting it to any certain problem?

3) Again, can you think of other resources/approaches that I could draw upon/utilize to scaffold the teaching of algebraic concepts to students? Furthermore, do you have any resources that I could use to scaffold my learning about algebra (old texts, electronic sources, etc.)?

4) Does the data that I have collected help you to understand my case and/or believe what I have written? If not, what else would you like to see and why would that be useful? *(please be frank about your impressions)*

V. Attachments

1) Artifacts- *to help you see what I am doing*
    a. Instructional Tasks: Math Menus 3.3 & 4.1, "Exploring Shapes Sheet" *(This last sheet is two sided. I filled in one side with values and encourage you to try responding to the questions before you look at the student samples)*

    b. Student work samples:
        i. **Math Menu 3.3**
        (notice the continued use of a variety of representations)
        ii. **Math Menu 4.1**
        (notice how students are looking for "rules", "patterns", and how "one solution leads to another")
        iii. **"Exploring Shapes Sheet"**
        (notice the amazing variety of equations [I will bring shapes to show how these equations work], but also notice their lack of explanation/labeling/grouping…)
    c. Assessment experiment

2) Selected readings- *to support understanding of what I am doing instructionally*
    a. Samples from the "Intermath" website showing problem type investigations of shapes that could lead to the development of equations
    b. Samples from various websites that have supported my own understanding of formulas that relate to geometry
    c. The Algebra Standard from the NCTM Principles and Standards (2000) and a nice explanation of three sorts of work (generalizing, globalizing, and extending) that students can be doing when they explore algebra (thanks Jody)
    d. Examples of tasks from two of the texts that are being piloted by the math committee (ready or not we will be moving toward this sort of "rule of 3" exploration with our students when we adopt a new math textbook)