Enriching the Mathematical Experiences of All Students
Through the "Rule of 3,4,….12?"
TRG Rough Case
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I. Rationale

Simply put the Rule of 3 states that every topic in mathematics should be presented numerically, algebraically, and geometrically (I understand that this last term refers for the most part to graphic representation) *(see attached Hughes reading)*. I have multiple reasons for working to understand the instructional and learning implications of the Rule of 3. First, there are several parents who inquired, in some cases forcefully, about what would be done to challenge their highly capable children. The students in fifth grade this year do indeed represent probably one of the most pronounced splits between highly capable and struggling students (with very few students in the middle) that we have had in quite some time. This seems to accentuate the needs of both groups. Exploring this strategy may be a way to address the needs of the mathematically advanced students, while not creating a whole new curriculum for them. Second, for the past few years I have noticed that the mathematical support that students are able to give for their problem solving solutions is often minimal or altogether lacking. Since I have never done a TRG case in mathematics, this topic provides a unique opportunity to match a real need with a personal change of pace. Third, in some senses given the direction of our district this year a case into extending learning experiences for otherwise "MEAP" ready students might seem like I am pointing my arrow in the wrong direction. However, since the tests that students take are so heavily grounded in multiple representations and since I am attempting to make my work on the Rule of 3 accessible to most of my students my work might seem more justifiable *(see attached NCTM standard, MI standard, and released MEAP item).*

II. **Information on Instructional Practices**

Each week I give a set of five math problems to my students that center on the math content that will be taught during the week *(see instructional examples)*. Students choose to complete a minimum of three problems from the "menu" and work independently, with small groups, and through large group discussions to solve these problems. They self assess and then I evaluate their performances using a scoring chart *(see attached assessment piece).* For the most part menus are perceived as challenging as they call for students to not only solve the problems but to share strategies and support their thinking in a variety of ways. Through small and whole group lessons I attempt to show students how the Rule of 3 can provide insight into their work. I am trying to show how these different representations are useful in problem solving, not just a novelty or busy work. I am encouraging students to think about the ways that these representations help them to see different aspects of the problem or understand the possibilities of the solutions better. In addition to the use of menus, I am also beginning to use text pilot materials from the math committee to teach students about the use of variables *(see instructional examples)*. On the sheets from these texts students learn to create simple equations and use them to find and explain solutions. I have also created some tasks of my own meant to engage students in thinking about variables.

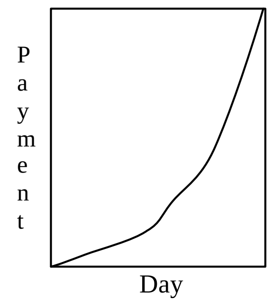
Even though I have worked on this case for a very short period of time at this point I am already starting to find out a few things. First, even though a colleague and I designed the entire set of menus that we now use, I know only some of the graphic connections and almost none of the algebraic connections to the problems in the menu. This was becoming a problem because I was pushing every problem into the Rule of 3 format and found that some seemingly simple problems were not very easily represented with algebra or in a graphic. As you look at these examples, note that Hughes says each *topic* should be presented this way not each problem!

Troublesome Algebra Example: A person is paid one cent the first day, two cents the second day, 4 cents the third day, 8 cents the fourth day,…for a total of 10 days. How much money does he make?

Numerically: .01 + .02 + .04 +. 08 + .16 + .32 + .64 + 1.28 + 2.56 + 5.12= $10.23

Algebraically: Daily cost- 2^{n-1} x .01 !!!!!!

Graphically:

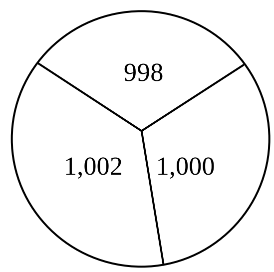


Troublesome Graphic Example: Three consecutive even numbers have a sum of 3,000. What are those numbers?

Numerically: 998 + 1,000 + 1,002 = 3,000

Algebraically: a + (a+2) + (a+4)= 3,000

Graphically!!!!:



Second, as you can begin to see in the work samples students are using a variety of representations, the pushed against my narrow vision of what "graphically" could mean *(see attached work samples).* However, even as students began to use more graphic representations, they were not always terribly mathematical or a solid basis for solving the problems posed on the menus. Third, I also don't know how students learn to use variables. Once students started to think about algebra, they wanted every number and even every operation to have its own letter. This did not lead to very useful equations, thereby possibly skewing student perceptions of algebra. Fourth, it turns out that the Rule of 3 has been augmented in the second edition of the Hughes book to include verbalizing in multiple ways *(see Villanova web article)*. Even though it might more properly be called something like the rule of 4 or the rule of 12 when considering all of the interrelations of the ideas, I will keep referring to this approach as grounded in the Rule of 3.

To study student's use of the Rule of 3 and my instruction related to it I have been video taping lessons and looking at student menus. The video works well in whole group discussion, but is usually not able to catch the small group interactions around the various representations. In the video I can see how the comments made by students; the solutions that they are willing to share; and their attention indicate their "uptake" of Rule of 3 ideas. With menu work I am looking for meaningful use of Rule of 3 ideas. This means that students will use the different representations more regularly, but also use them to provide more substantial support their strategies, solutions and thinking.

III. Questions for the Group

1) a. Does the way that I have explained my use of the Rule of 3 seem like it would be graspable by most students in some form?

   b. Can you think of other resources/approaches that I could draw upon/utilize to scaffold the teaching of algebraic concepts to students? Furthermore, do you have any resources that I could use to scaffold my learning about algebra (old texts, electronic sources, etc.)?

2) How could I adjust my assessment sheet or practices related to that sheet to emphasize *the meaningful use* of multiple representations? How could I use this or other data to collect information about student use and understanding of the Rule of 3?

3) Given the current imperatives in our district, is the exploration of the issues I have laid out in this case warranted or defensible? *(I really am hoping for frank opinions related to this question)*

IV. Attachments

1) Artifacts- *to help you see what I am doing*
   a. Instructional Tasks: Math Menu 2.3, Everyday Math "Number Stories" and "Mystery Number"
   b. Student work samples focusing on the "Frog Problem" (menu problem 1) and Mystery Number
   c. Assessment piece

2) Selected readings- *to help provide support for what I am doing instructionally*
   a. "Preface" from *Calculus* by Deborah Hughes-Hallett, et al. (1994) and "The Rule of Four" from a Villanova website
   b. The Representation Standard from the NCTM Principles and Standards (2000)
   c. Michigan Benchmarks on number, algebra, and data & statistics
   d. Released MEAP items with multiple representations
   e. "The Nature of Classroom Tasks" taken from *Making Sense* by James Hiebert et al. (1997)